On a new type of information processing for efficient management of complex systems

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1 Introduction

It is a challenge to manage complex systems efficiently without confronting NP-hard problems. To address the situation we use the description of complex systems in terms of self-organization processes of prime integer relations [1] and suggest that the use of the processes might open a way to a new type of information processing.

2 The hierarchical network of prime integer relations

The description is realized through the unity of two equivalent forms, i.e., arithmetical and geometrical [1], [2].

2.1 The arithmetical form

In the arithmetical form a complex system is characterized by hierarchical correlation structures built in accordance with self-organization processes of prime integer relations. As each of the correlation structures is ready to exercise its own scenario and there is no mechanism specifying which of them is going to take place, an intrinsic uncertainty about the complex system exists. At the same time, the information about the correlation structures can be used to evaluate the probability of an observable to take each of the measurement outcomes. Therefore, the arithmetical form of the description provides the statistical information about a complex system.

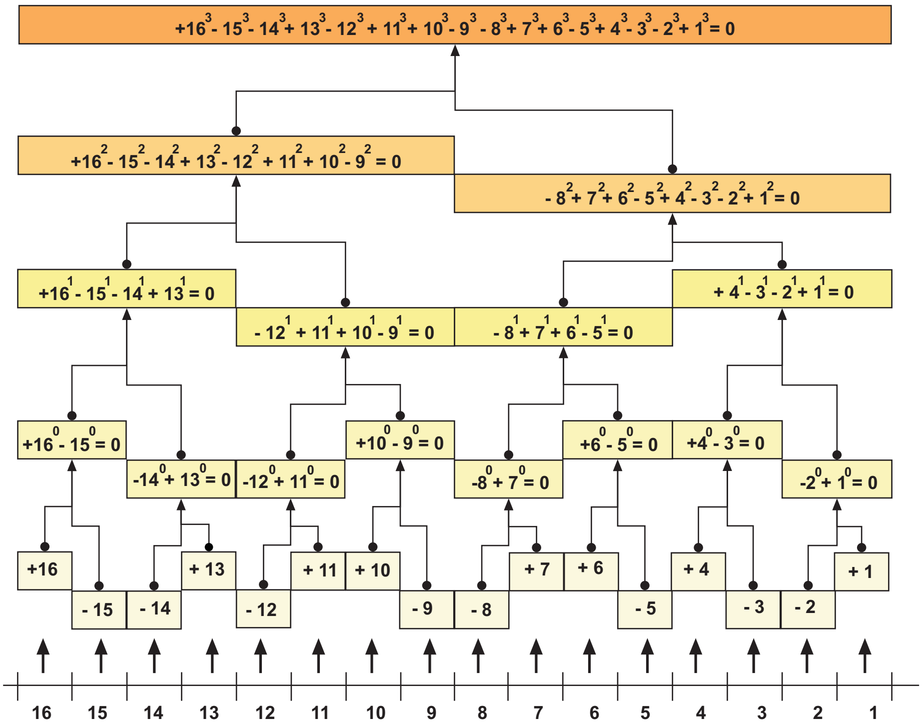

Figure 1: The figure shows a hierarchical structure of prime integer relations defined as a result of a self-organization process starting at level 0 with integers 16, 13, 11, 10, 7, 6, 4, 1 in the "positive state" and integers 15, 14, 12, 9, 8, 5, 3, 2 in the "negative". The process is controlled by arithmetic and cannot progress to level 5, because arithmetic determines that $+16^4 - 15^4 - 14^4 + 13^4 - 12^4 + 11^4 + 10^4 - 9^4 - 8^4 + 7^4 + 6^4 - 5^4 + 4^4 - 3^4 - 2^4 + 1^4 \neq 0$. This hierarchical structure can specify a correlation structure of a system made up of 16 elementary parts P_i whose changes Δx_i of the observables $x_i, i = 1, ..., 16$ in their reference frames are given by the sequence $\Delta x_1 ... \Delta x_{16} = +1 - 1 - 1 + 1 - 1 + 1 + 1 - 1 - 1 + 1 + 1 - 1 + 1 - 1 - 1 + 1$.

The arithmetical form reveals nonlocal correlations without reference to signals as well as the distances and local times of the parts. Thus, the arithmetical form suggests that parts of a complex systems may be far apart in space and time and yet remain interconnected with instantaneous effect on each other, but no signalling. Namely, if a correlation structure of a system is selected and some parts are specified, then through the prime integer relations in control of the correlation structure the other parts are immediately defined.

Through the arithmetical form of the description we become aware of the hierarchical network, i.e., a set of mutually self-consistent elements built by all self-organization processes of prime integer relations. Arithmetic ensures that not even a minor change can be made to any element of the network (Figure 1 gives an illustration). It is helpful to think that self-organization processes of prime integer relations take place in the hierarchical network as they compose more and more complex structures.

The hierarchical network of prime integer relations is a causal structure. As parts of a system change and the prime integer relations cause the other parts to change accordingly, an event takes place. Once the changes have been realized, the event is fixed in space and time with respect to the reference frames of the parts. For the parts the effect of the event has not necessarily be the same, but for each part it is appropriately determined by the prime integer relations. However, the prime integer relations at work for a system have no causal power to effect systems controlled by separate prime integer relations. As a result, information about the systems is blocked for the observers of the system.

2.2 The geometrical form

Specified by two parameters $\varepsilon > 0$ and $\delta > 0$ the geometrical form arises as the self-organization processes of prime integer relations find isomorphic realization in terms of transformations of two-dimensional geometrical patterns [1]. As a result, hierarchical structures of prime integer relations defining the correlation structures of a complex system become equivalently represented by hierarchical structures of geometrical patterns determining the dynamics of the system. In particular, quantities of the geometrical patterns can be used to describe quantities of the system [2].

Figure 2 illustrates the correspondence between the two forms by showing a hierarchical structure of geometrical patterns, which for given $\varepsilon > 0$ and $\delta > 0$ is isomorphic to the hierarchical structure of prime integer relations depicted in Figure 1. A scale invariant property and a renormalization group transformation come to our attention, as we consider the connection between a geometrical pattern and the geometrical patterns it is made of. Although the geometrical patterns are not triangles at level $l, l = 2, 3, 4$, yet the boundary curve as the graph of the function $\Psi_1^{[l]}$ is such that the area S_l of a geometrical pattern can be simply given, as if it were a triangle, by $S_l = W_l H_l/2$, where W_l and H_l are its width and height. The renormalization group transformation at level 4 defines $\varepsilon' = 2^3\varepsilon, \delta' = \varepsilon^3\delta$ and uses a coarse-grained procedure replacing the geometrical pattern made up of the 8 geometrical patterns at level 1 by their

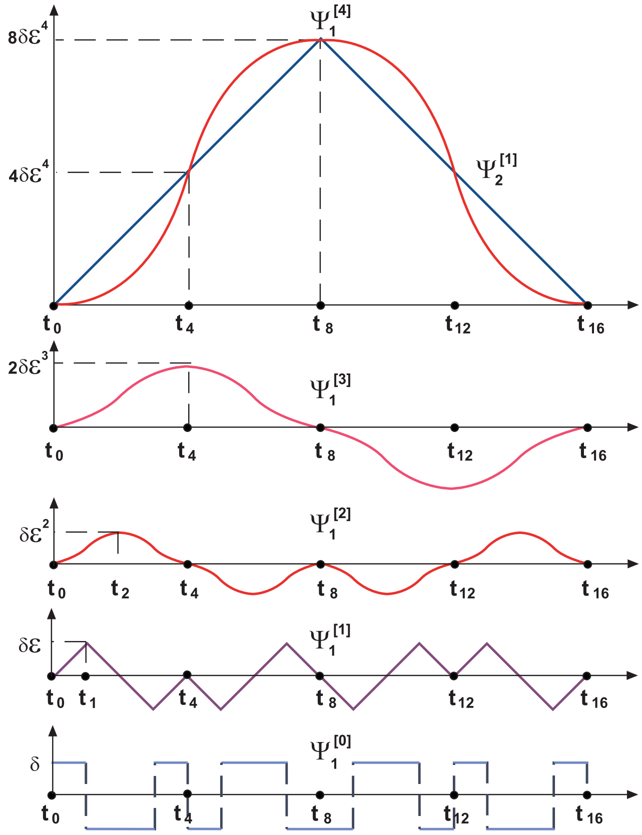

Figure 2: The figure shows a hierarchical structure of geometrical patterns, which for given ε and δ is isomorphic to the hierarchical structure of prime integer relations shown in Figure 1. Under the integration of the function $\Psi_1^{[l]}, \Psi_1^{[l+1]}(t_0) = 0, l = 0, 1, 2, 3$ the geometrical patterns at the level l form the geometrical patterns at the higher level $l + 1$. The width W_l of a geometrical pattern at level $l, l = 1, ..., 4$ equals the sum $W_l = W_{l-1,left} + W_{l-1,right} = 2W_{l-1}$ of the widths of the two geometrical patterns it is made up of, where $W_{l-1,left}$ and $W_{l-1,right}$ are the widths of the left and right geometrical patterns and $W_{l-1,left} = W_{l-1,right} = W_{l-1}$ as each geometrical pattern at level $l - 1$ has the same width $W_{l-1} = 2^{l-1}\varepsilon$ and $t_i = \varepsilon i, i = 0, 1, ..., 16$. The height H_l of the geometrical pattern equals $H_l = S_{l-1}$, where S_{l-1} is the area of each of the geometrical patterns it is composed of. The width W_l and the height H_l can specify length scales of the level $l, l = 0, 1, ..., 4$.

enlarged version with the boundary curve as the graph of the function $\Psi_2^{[1]}$. Each of the geometrical patterns at level 1 is elementary in the sense that it is fully specified by a prime integer relation made up of two integers and they have no further internal structure. The width $2\varepsilon'$ and the height $\delta'\varepsilon'$ of the renormalized geometrical pattern at level 4 are given in terms of ε' and δ' in the same way as the width 2ε and the height $\delta\varepsilon$ of a geometrical pattern at level 1 are given in terms of ε and δ. The two geometrical patterns at level 4 have the same width, height and area $\int_{t_0}^{t_{16}} \Psi_1^{[4]}(t)dt = \int_{t_0}^{t_{16}} \Psi_2^{[1]}(t)dt$, but the lengths of their boundary curves are different.

The two equivalent forms shows the dynamics and the structure of a complex system as two sides of the same entity that preserves the system as a whole. At one side, the dynamics of a complex system is determined to produce spacetime patterns of the parts to fit precisely into the geometrical patterns. Through the isomorphism at the other side the prime integer relations then provide the relationships of the complex system. If the spacetime patterns do not fit even slightly, then one or more of the relationships provided by the prime integer relations are not in place and the complex system collapses.

To measure the complexity of a system in terms of self-organization processes of prime integer relations a concept of complexity, called the structural complexity, is introduced [1]. Starting with the integers, the self-organization processes of prime integer relations progress to different levels and thus produce a hierarchical complexity order. In particular, the higher the level self-organization processes progress to, the greater is the structural complexity of a corresponding system. In our description systems can be compared in terms of complexity by using two equivalent forms: in structure - by the hierarchical structures of prime integer relations and in dynamics - by the hierarchical structures of geometrical patterns.

Remarkably, based on integers and controlled by arithmetic only self-organization processes of prime integer relations can describe complex systems by information not requiring further explanation or simplification.

3 Testing a new medium for information processing

The correlation structures of a complex system contain information about the parts. By changing some parts the information can be processed as the other parts change in accordance with the prime integer relations. The capacity to process information should depend on the correlation structures and vary as the self-organization processes of prime integer relations change in the hierarchical network.

We suggest the hierarchical network of prime integer relations as a new medium for information processing and consider whether it could deliver significant advantages. In particular, we are interested whether a problem could be efficiently solved by a computing system built specially for the problem by using self-organization processes of prime integer relations. To make the efficient solution possible the hierarchical network would be required to have remarkable navigating properties. It would be beneficial if the performance of a computing system were a concave function of its structural complexity. With this guide in place the performance global maximum could be efficiently found. It would be also helpful if the structural complexities of the system and the problem at the global maximum were related through an optimality condition.

The optimality condition might be interpreted as follows: if arithmetical interdependencies emerging between a computing system and a problem form in the hierarchical network a new building block at the highest possible level, the optimal performance takes place. Or, in other terms, a computing system finds the solution to a problem, once arithmetic interdependencies emerging between the system and the problem provide a channel to obtain the desired information.

Since the correlation structures of a system are completely determined by prime integer relations, which are equivalent to two-dimensional geometrical patterns, the entropy of the system, measuring its information content, can be connected with the areas of the two-dimensional patterns. Thus, in our approach there is a general connection between entropy and area.

Computational experiments have been conducted [3] to investigate whether: 1. the performance of an optimization algorithm for an NP-hard problem could behave as a concave function of the algorithm's structural complexity; 2. the structural complexities of the algorithm and the problem at the performance global maximum could be related through an optimality condition.

In particular, an optimization algorithm $\mathcal{A}$, as a complex system, of N computational agents minimizing the average distance in the travelling salesman problem (TSP) is developed. The agents work in parallel and start in the same city by choosing the next city at random. Then an agent at each step visits the next city by using one of the two strategies: a random strategy or the greedy strategy. In the solution of a problem with n cities the state of the agents at step $j, j = 1, ..., n-1$ can be described by a binary sequence $s_j = s_{1j}...s_{Nj}$, where $s_{ij} = +1$, if agent $i, i = 1, ..., N$ uses the random strategy and $s_{ij} = -1$, if the agent i uses the greedy strategy. The dynamics of the complex system is realized as the agents step by step choose their strategies and can be encoded by an $N \times (n-1)$ binary strategy matrix $S = \{s_{ij}, i = 1, ..., N, j = 1, ..., n-1\}$.

We try to change the structural complexity of the algorithm $\mathcal{A}$ monotonically in forcing the system to make the transition from regular behaviour to chaos by period-doubling. To control the system in this transition a parameter $v, 0 \leq v \leq 1$ is introduced. It specifies a threshold point dividing the interval of the current distances passed by the agents into two parts, i.e., successful and unsuccessful. This information is provided for an optimal if-then rule [1] that each agent uses to choose the next strategy. The rule relies on the Prouhet-Thue-Morse (PTM) sequence $+1-1-1+1-1+1+1-1$ ... and has the following description:

1. if the last strategy is successful, continue with the same strategy.

2. if the last strategy is unsuccessful, consult PTM generator which strategy to use next.

Remarkably, for each problem p tested from a class $\mathcal{P}$ it has been found that the performance of the algorithm $\mathcal{A}$ indeed behaves as a concave function of the control parameter with the only global maximum at a value $v^*(p)$. The global maximums $\{v^*(p), p \in \mathcal{P}\}$ are of interest to probe whether the structural complexities of the algorithm $\mathcal{A}$ and the problem are related through an optimality condition. For this purpose strategy matrices $\{S(v^*(p)), p \in \mathcal{P}\}$ corresponding to the global maximums $\{v^*(p), p \in \mathcal{P}\}$ are used and the structural complexities of the algorithm $\mathcal{A}$ and a problem p are approximated as follows. The structural complexity $\mathbf{C}(A(p))$ of the algorithm $\mathcal{A}$ is approximated by the quadratic trace

$$C(\mathcal{A}(p)) = \frac{1}{N^2} tr(V^2(v^*(p))) = \frac{1}{N^2} \sum_{i=1}^{N} \lambda_i^2$$

of the variance-covariance matrix $V(v^*(p))$ obtained from the strategy matrix $S(v^*(p))$, where $\lambda_i, i = 1, ..., N$ are the eigenvalues of $V(v^*(p))$. The structural complexity $\mathbf{C}(p)$ of the problem p is approximated by the quadratic trace

$$C(p) = \frac{1}{n^2} tr(M^2(p)) = \frac{1}{n^2} \sum_{i=1}^{n} (\lambda_i')^2$$

of the normalized distance matrix $M(p) = \{d_{ij}/d_{max}, i, j = 1, ..., n\}$, where $\lambda_i', i = 1, ..., n$ are the eigenvalues of $M(p)$, d_{ij} is the distance between cities i and j and d_{max} is the maximum of the distances.

To reveal the optimality condition the points with the coordinates $\{x = C(p), y = C(\mathcal{A}(p)), p \in \mathcal{P}\}$ are considered. The result has been indicative of a linear relationship between the structural complexities and thus suggests an optimality condition of the algorithm $\mathcal{A}$ [3]:

If the algorithm $\mathcal{A}$ demonstrates the optimal performance for a problem p, then the structural complexity $C(\mathcal{A}(p))$ of the algorithm $\mathcal{A}$ is in the linear relationship $C(\mathcal{A}(p)) = 0.67C(p) + 0.33$ with the structural complexity $C(p)$ of the problem p.

The computational results point that classical optimization algorithms may be designed comparable to efficient quantum algorithms.

Quantum computation relies on the practical use of entanglement, whose understanding is still limited and sensitivity challenges the development of relevant technologies. In a TSP quantum algorithm the wave function would be evolved to maximize through the amplitudes the probability of the shortest routes to be measured. However, there is no general direction known for the evolution to take in order to make the quantum algorithm efficient. In this regard the majorization principle seems to play an important role [4]. It provides a local navigation, but without information about the whole landscape.

In our approach the nonlocal correlations are known from the origin in the self-organization processes of prime integer relations. The key question is whether this knowledge could provide computational resources comparable to quantum computation. In this paper we focus on whether such a resource could be obtained from the nonlocal correlations when projected into in the classical reality, but with the order preserved. If that could be possible, then the order of the self-organization processes would establish a general direction for efficient computation. Remarkably, the experiments raise the possibility that the performance landscape becomes concave, if the evolution goes in this direction.

In the experiments we use the parameter to control the correlation structures of the computing system with their consequences observed in the routes taken by the agents. To help in associations with the quantum case the average distance for a value v of the parameter can be written as

$$\bar{D} = (\gamma_{1,...,n-1}(v)d([1,...,n-1>) + ... + \gamma_{n-1,...,1}(v)d([n-1,...,1>))/N,$$

where $\gamma_{i_1,...,i_{n-1}}$ is the number of the agents followed the route $[i_1,...,i_{n-1}>$ $, d([i_1,...,i_{n-1}>)$ is its distance and the n cities are labelled by $0,1,...,n-1$ with the initial city by 0. The interpretation of the coefficient $\gamma_{i_1,...,i_{n-1}}$ as the probability of the route $[i_1,...,i_{n-1}>$ suggests that the minimization of the average distance considered in the algorithm $\mathcal{A}$ can be connected with the maximization of the probability of the shortest route considered in TSP quantum algorithms. Significantly, the experiments have shown that in the case of the algorithm $\mathcal{A}$ the maximization of the probability turns out to be a one-dimensional concave optimization. In its course the computing system got more complex or simpler until the global maximum is reached and the structural complexities of the computing system and the problem become related through the optimality condition. More connections might arise if the wave function could be involved in the description of the correlation structures.

We note that the algorithm $\mathcal{A}$ shares a common feature with Shor's algorithm, which also relies on the PTM sequence [5].

4 Conclusions

We have suggested that self-organization processes of prime integer relations could be used for information processing. Advantages might be expected as the correlation structures of the system processing the information could be built specially for the problem by using such self-organization processes. The information processing would be distinctive, because the processes can define the correlation structures without references to the distances, local times and signals between the parts. Remarkably, the self-organization processes of prime integer relations can be equivalently represented as transformations of two-dimensional geometrical patterns determining the dynamics of the system and revealing its structural complexity in the information processing.

Computational experiments testing competitive advantages of the information processing to manage complex systems efficiently have been presented. They raise the possibility of an optimality condition of complex systems: if the structural complexity of a system is in a certain relationship with the structural complexity of a problem, then the system demonstrates the optimal performance for the problem.

Importantly, the experiments indicate that the performance of a complex system may behave as a concave function of the structural complexity. Therefore, once the structural complexity could be controlled as a single entity, the optimization of a complex system would be potentially reduced to a one-dimensional concave optimization irrespective of the number of variables involved its description. This might open a way to a new type of information processing for efficient management of complex systems.

Bibliography

[1] Victor KOROTKIKH, *A Mathematical Structure for Emergent Computation*, Kluwer Academic Publishers, Dordrecht/Boston/London (1999).

[2] Victor KOROTKIKH and Galina KOROTKIKH, *On an Irreducible Theory of Complex Systems*, InterJournal of Complex Systems, 2006, 1751; arXiv:nlin/0606023.

[3] Victor KOROTKIKH, Galina KOROTKIKH and Darryl BOND, *On Optimality Condition of Complex Systems: Computational Evidence*, arXiv:cs/0504092.

[4] Roman ORUS, Jose LATORRE and Miguel MARTIN-DELGADO, *Systematic Analysis of Majorization in Quantum Algorithms*, arXiv:quant-ph/0212094.

[5] Krishnendu MAITY and Arul LAKSHMINARAYAN, *Quantum Chaos in the Spectrum of Operators Used in Shor's Algorithm*, arXiv:quant-ph/0604111.