

# On a new type of information processing for efficient management of complex systems

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## 1 Introduction

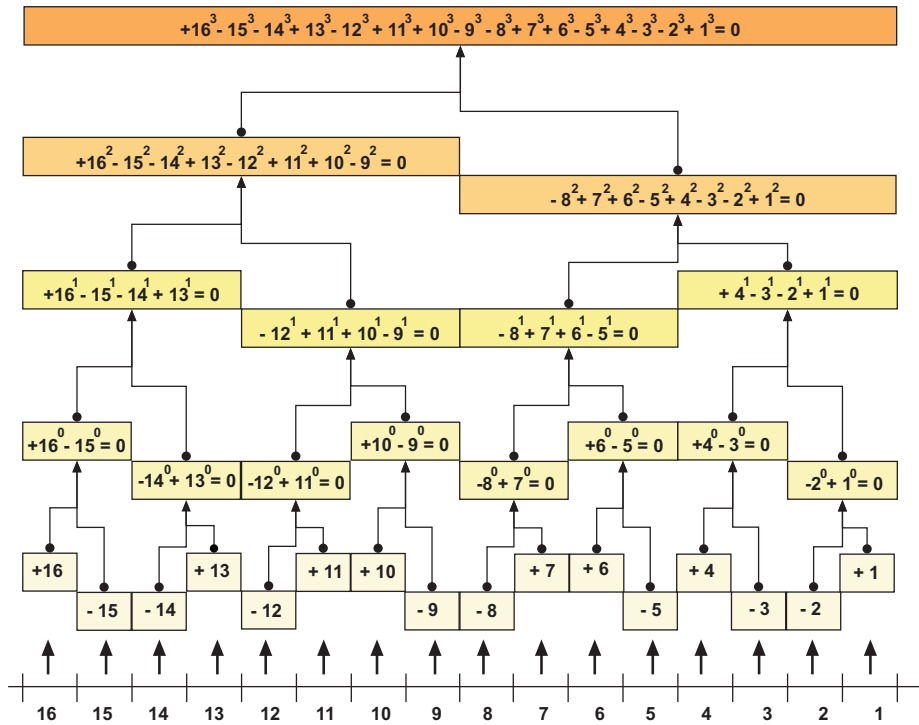
It is a challenge to manage complex systems efficiently without confronting NP-hard problems. To address the situation we consider the description of complex systems in terms of self-organization processes of prime integer relations [1] and suggest to use the processes for information processing. In particular, we propose that for a given problem self-organization processes of prime integer relations could be used to efficiently build a computing system demonstrating the optimal performance for the problem.

## 2 The hierarchical network of prime integer relations

The description is realized through the unity of two equivalent forms, i.e., arithmetical and geometrical [1], [2].

## 2.1 The arithmetical form of the description

In the arithmetical form a complex system is characterized by hierarchical correlation structures built in accordance with self-organization processes of prime integer relations. As each of the correlation structures is ready to exercise its own scenario and there is no mechanism specifying which of them is going to take place, an intrinsic uncertainty about the complex system exists. At the same time, the information about the correlation structures can be used to evaluate the probability of an observable to take each of the measurement outcomes. Therefore, the arithmetical form of the description provides the statistical information about a complex system.



**Figure 1:** The figure shows a hierarchical structure of prime integer relations defined as a result of a self-organization process starting at level 0 with integers 16, 13, 11, 10, 7, 6, 4, 1 in the "positive state" and integers 15, 14, 12, 9, 8, 5, 3, 2 in the "negative". The process is controlled by arithmetic and cannot progress to level 5, because arithmetic determines that  $+16^4 - 15^4 - 14^4 + 13^4 - 12^4 + 11^4 + 10^4 - 9^4 - 8^4 + 7^4 + 6^4 - 5^4 + 4^4 - 3^4 - 2^4 + 1^4 \neq 0$ . This hierarchical structure can specify a correlation structure of a system made up of 16 elementary parts whose changes  $\Delta x_i$  of the observables  $x_i, i = 1, \dots, 16$  in their reference frames are given by the sequence  $\Delta x_1 \dots \Delta x_{16} = +1 - 1 - 1 + 1 - 1 + 1 + 1 - 1 - 1 + 1 + 1 - 1 + 1 - 1 - 1 + 1$ .

The arithmetical form reveals nonlocal correlations without reference to sig-

nalling as well as the distances and local times of the parts. Thus, the arithmetical form suggests that parts of a complex systems may be far apart in space and time and yet remain interconnected with instantaneous effect on each other, but no signalling. Namely, if a correlation structure of a system is selected and some parts are specified, then through the prime integer relations in control of the correlation structure the other parts are immediately defined.

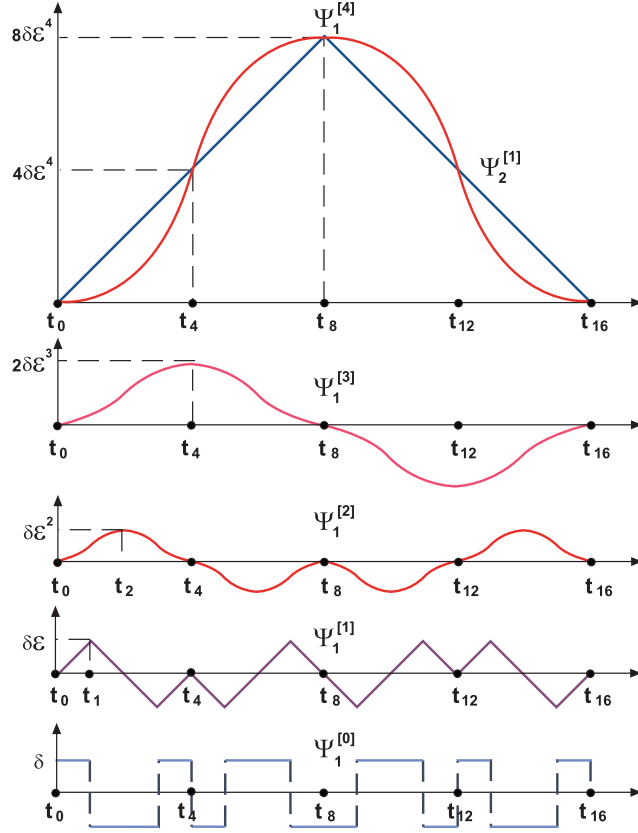
Through the arithmetical form of the description we become aware of the hierarchical network of prime integer relations, i.e., a set of mutually self-consistent elements built by the processes. Arithmetic ensures that not even a minor change can be made to any element of the network (Figure 1). The hierarchical network has a special order and as self-organization processes progress in the direction of this order more complex systems are composed.

The hierarchical network of prime integer relations is a causal structure. As parts of a system change and the prime integer relations cause the other parts to change accordingly, an event takes place. Once the changes have been realized, the event is fixed in space and time with respect to the reference frames of the parts. The effect of the event has not necessarily be the same for the parts, but for each part it is appropriately determined by the prime integer relations. However, the prime integer relations at work for a system have no causal power to effect systems controlled by separate prime integer relations. As a result, information about the systems is blocked for the observers of the system.

## 2.2 The geometrical form of the description

Specified by two parameters  $\varepsilon > 0$  and  $\delta > 0$  the geometrical form arises as the self-organization processes of prime integer relations find isomorphic realization in terms of transformations of two-dimensional geometrical patterns [1]. As a result, hierarchical structures of prime integer relations defining the correlation structures of a complex system become equivalently represented by hierarchical structures of geometrical patterns determining the dynamics of the system and revealing its complexity. The quantitative description of the system turns out to be about the description of these geometrical patterns [1], [2].

Figure 2 shows a hierarchical structure of geometrical patterns, which for given  $\varepsilon > 0$  and  $\delta > 0$  is isomorphic to the hierarchical structure of prime integer relations depicted in Figure 1. A scale invariant property and a renormalization group transformation come to our attention, as we consider the connection between a geometrical pattern and the geometrical patterns it is made of. Although the geometrical patterns are not triangles at a level  $l, l = 2, 3, 4$ , yet the boundary curve, as the graph of the function  $\Psi_1^{[l]}$ , is such that the area  $S_l$  of a geometrical pattern can be simply calculated, as if it were a triangle, by  $S_l = W_l H_l / 2$ , where  $W_l$  and  $H_l$  are its width and height. The renormalization group transformation at level 4 defines  $\varepsilon' = 2^3 \varepsilon, \delta' = \varepsilon^3 \delta$  and uses a coarse-grained procedure replacing the geometrical pattern made up of 8 geometrical patterns at level 1 by their enlarged version with the boundary curve as the graph of the function  $\Psi_2^{[1]}$ . Each of the geometrical patterns at level 1 is elemen-



**Figure 2:** Under the integration of the function  $\Psi_1^{[l]}$ ,  $\Psi_1^{[l+1]}(t_0) = 0$ ,  $l = 0, 1, 2, 3$  the geometrical patterns at a level  $l$  form the geometrical patterns at the higher level  $l + 1$ . The width  $W_l$  of a geometrical pattern at a level  $l$ ,  $l = 1, \dots, 4$  equals the sum  $W_l = W_{l-1, \text{left}} + W_{l-1, \text{right}} = 2W_{l-1}$  of the widths of the two geometrical patterns it is made of, where  $W_{l-1, \text{left}}$  and  $W_{l-1, \text{right}}$  are the widths of the left and right geometrical patterns and  $W_{l-1, \text{left}} = W_{l-1, \text{right}} = W_{l-1}$  as each geometrical pattern at the level  $l - 1$  has the same width  $W_{l-1} = 2^{l-1}\varepsilon$  and  $t_i = \varepsilon i$ ,  $i = 0, 1, \dots, 16$ . The height  $H_l$  of the geometrical pattern equals  $H_l = S_{l-1}$ , where  $S_{l-1}$  is the area of each of the geometrical patterns it is composed of. The width  $W_l$  and height  $H_l$  can specify length scales of the level  $l$ ,  $l = 0, 1, \dots, 4$ .

tary in the sense that it is fully specified by a prime integer relation made up of two integers, which have no internal structure. The width  $2\varepsilon'$  and the height  $\delta'\varepsilon'$  of the renormalized geometrical pattern at level 4 are given in terms of  $\varepsilon'$  and  $\delta'$  in the same way as the width  $2\varepsilon$  and the height  $\delta\varepsilon$  of a geometrical pattern at level 1 are given in terms of  $\varepsilon$  and  $\delta$ . The two geometrical patterns at level 4 have the same width, height and area  $\int_{t_0}^{t_{16}} \Psi_1^{[4]}(t)dt = \int_{t_0}^{t_{16}} \Psi_2^{[1]}(t)dt$ , except the lengths of their boundary curves (Figure 2).

The equivalence of the forms unites the dynamics and structure of a complex system. Based on the fact that arithmetic behind a prime integer relation makes it sensitive even to a minor change of coefficients, the equivalence introduces a new principle. Notably, a breaking of a prime integer relation leads to a collapse of a corresponding system, as some of the relationships disappear. The principle states that the dynamics of the parts is determined to produce precisely the geometrical patterns corresponding to the prime integer relations, which provide the correlation structures of the system. If the dynamics is even slightly different, then some of the relationships are not in place and the system collapses.

The quantity of a system characterized by the length of the boundary curve of a geometrical pattern may define a fine-tuned parameter. The parameter cannot be changed even a bit, because it provides the link with the boundary curve and the prime integer relation. Successive digits of a parameter may appear at random, yet beyond the randomness the description recognizes the possibility of determinism. Indeed, as long as the parameter corresponds to prime integer relations, each digit must stand as it is and not even slightly different. This helps to realize that prime integer relations determine parameters of a complex system completely.

To measure the complexity of a system in terms of self-organization processes of prime integer relations a concept of complexity, called the structural complexity, is introduced [1]. Starting with the integers, the self-organization processes of prime integer relations progress to different levels and thus produce a hierarchical complexity order. The higher the level self-organization processes progress to, the greater is the structural complexity of a corresponding system. The description allows to compare complexities of systems in terms of structure - by hierarchical structures of prime integer relations and dynamics - by hierarchical structures of geometrical patterns.

Remarkably, based on integers and controlled by arithmetic only self-organization processes of prime integer relations can describe complex systems by information not requiring further simplification.

### **3 Testing a new medium for information processing**

The correlation structures of a complex system contain information about the parts. By changing some parts the information can be processed as the other parts change in accordance with the prime integer relations. This shows the importance of self-organization processes of prime integer relations for information processing. Namely, for a given problem they could be used to efficiently build the correlation structures of a computing system in information processing demonstrating the optimal performance for the problem.

As a result, we suggest the hierarchical network of prime integer relations as a new medium for information processing and investigate its navigating properties. It would be important if for a given problem the performance of a system

could behave as a concave function of its structural complexity. Guided by this property the performance global maximum could be efficiently found. It would be also beneficial, if at the global maximum the structural complexities of the system and the problem could be related through an optimality condition.

The optimality condition could be interpreted as follows: if through arithmetical interdependencies emerging between a computing system and a problem a new building block is formed at the highest possible level, then the optimal performance takes place. Or, in other terms, a computing system finds the solution to a problem, once arithmetic interdependencies emerging between them provide a channel to obtain the desired information.

It is worth to note that since the correlation structures of a system are completely determined by prime integer relations, which are equivalent to two-dimensional geometrical patterns, the entropy of the system, measuring its information content, can be connected with the areas of the patterns. Thus, in our approach there is a general connection between entropy and area.

Computational experiments have been conducted [3] to test the navigating properties. In particular, an optimization algorithm  $\mathcal{A}$ , as a complex system, of  $N$  computational agents minimizing the average distance in the travelling salesman problem (TSP) is developed. The agents work in parallel and starting in the same city they choose the next city at random. At each step an agent relies on one of two strategies: random or greedy. In the solution of a problem with  $n$  cities the state of the agents at step  $j, j = 1, \dots, n - 1$  can be described by a binary sequence  $s_j = s_{1j} \dots s_{Nj}$ , where  $s_{ij} = +1$ , if agent  $i, i = 1, \dots, N$  uses the random strategy and  $s_{ij} = -1$ , if the agent  $i$  uses the greedy strategy. Therefore, the dynamics of the complex system can be encoded by an  $N \times (n - 1)$  binary strategy matrix  $S = \{s_{ij}, i = 1, \dots, N, j = 1, \dots, n - 1\}$ .

We try to change the structural complexity of the algorithm  $\mathcal{A}$  monotonically by forcing the system to make the transition from regular behaviour to chaos by period-doubling. To control the system in this transition a parameter  $v, 0 \leq v \leq 1$  is introduced. It specifies a threshold point dividing the interval of current distances passed by the agents into two parts, i.e., successful and unsuccessful. This information is required for an optimal if-then rule [1] that each agent uses to choose the next strategy. The rule relies on the Prouhet-Thue-Morse (PTM) sequence  $+1 - 1 - 1 + 1 - 1 + 1 + 1 - 1 \dots$  and has the following description:

1. if the last strategy is successful, continue with the same strategy.
2. if the last strategy is unsuccessful, consult PTM generator which strategy to use next.

Remarkably, for each problem  $p$  tested from a class  $\mathcal{P}$  it has been found that the performance of the algorithm  $\mathcal{A}$  indeed behaves as a concave function of the control parameter with the only global maximum at  $v^*(p)$ . The global maximums  $\{v^*(p), p \in \mathcal{P}\}$  are of interest to probe whether the structural complexities of the algorithm  $\mathcal{A}$  and the problem are related through an optimality condition. For this purpose strategy matrices  $\{S(v^*(p)), p \in \mathcal{P}\}$  corresponding to the global maximums  $\{v^*(p), p \in \mathcal{P}\}$  are used and the structural complexities of the algorithm  $\mathcal{A}$  and a problem  $p$  are approximated as follows. The structural

complexity  $C(\mathcal{A}(p))$  of the algorithm  $\mathcal{A}$  is approximated by the quadratic trace

$$C(\mathcal{A}(p)) = \frac{1}{N^2} \text{tr}(V^2(v^*(p))) = \frac{1}{N^2} \sum_{i=1}^N \lambda_i^2$$

of the variance-covariance matrix  $V(v^*(p))$  obtained from the strategy matrix  $S(v^*(p))$ , where  $\lambda_i, i = 1, \dots, N$  are the eigenvalues of  $V(v^*(p))$ . The structural complexity  $C(p)$  of the problem  $p$  is approximated by the quadratic trace

$$C(p) = \frac{1}{n^2} \text{tr}(M^2(p)) = \frac{1}{n^2} \sum_{i=1}^n (\lambda'_i)^2$$

of the normalized distance matrix  $M(p) = \{d_{ij}/d_{max}, i, j = 1, \dots, n\}$ , where  $\lambda'_i, i = 1, \dots, n$  are the eigenvalues of  $M(p)$ ,  $d_{ij}$  is the distance between cities  $i$  and  $j$  and  $d_{max}$  is the maximum of the distances.

To reveal the optimality condition the points with the coordinates  $\{x = C(p), y = C(\mathcal{A}(p)), p \in \mathcal{P}\}$  are considered. The result indicates a linear relationship between the structural complexities and thus suggests an optimality condition of the algorithm  $\mathcal{A}$  [3]:

*If the algorithm  $\mathcal{A}$  demonstrates the optimal performance for a problem  $p$ , then the structural complexity  $C(\mathcal{A}(p))$  of the algorithm  $\mathcal{A}$  is in the linear relationship  $C(\mathcal{A}(p)) = 0.67C(p) + 0.33$  with the structural complexity  $C(p)$  of the problem  $p$ .*

The computational results point that it may be possible to design classical algorithms comparable to efficient quantum algorithms by using self-organization processes of prime integer relations. Quantum algorithms rely on the practical use of entanglement, whose sensitivity challenges the development of relevant technologies. In a TSP quantum algorithm the wave function would be evolved to maximize through the amplitudes the probability of the shortest route to be measured. However, there is no general direction known for the evolution to take in order to make the quantum algorithm efficient. In this regard the majorization principle provides a local navigation [4], but without information about the whole landscape.

While the nature of quantum entanglement is yet to be understood [5] in our approach the nonlocal correlations are known from their origin in the self-organization processes of prime integer relations. The question is whether this knowledge could be used to provide computational resources comparable to quantum computation. In this paper we focus on whether such a resource could be obtained from the nonlocal correlations used in classical computation, but with the order of the processes preserved. If it could be possible, then the order of the self-organization processes would establish a general direction for efficient computation. Remarkably, the experiments raise the possibility that the performance landscape becomes concave, if the evolution goes in this direction.

In the experiments we use the parameter to control the correlation structures of the computing system with their consequences observed in the routes taken by

the agents. To help in associations with the quantum case the average distance for a value  $v$  of the parameter can be written as

$$\bar{D} = (\gamma_{1,\dots,n-1}(v)d([1, \dots, n-1 >) + \dots + \gamma_{n-1,\dots,1}(v)d([n-1, \dots, 1 >))/N,$$

where  $\gamma_{i_1,\dots,i_{n-1}}$  is the number of the agents followed the route  $[i_1, \dots, i_{n-1} >$ ,  $d([i_1, \dots, i_{n-1} >)$  is its distance and the  $n$  cities are labelled by  $0, 1, \dots, n-1$  with the initial city by 0. The interpretation of the coefficient  $\gamma_{i_1,\dots,i_{n-1}}/N$  as the probability of the route  $[i_1, \dots, i_{n-1} >$  suggests that the minimization of the average distance considered in the algorithm  $\mathcal{A}$  can be connected with the maximization of the probability of the shortest route considered in a TSP quantum algorithm. Significantly, the experiments have shown that in the case of the algorithm  $\mathcal{A}$  the maximization of the probability turns out to be a one-dimensional concave optimization. In its course the computing system got more complex or simpler until the global maximum is reached and the structural complexities of the computing system and the problem become related through the optimality condition. More connections might arise if the wave function could be involved in the description of the correlation structures.

We note that the algorithm  $\mathcal{A}$  shares a common feature with Shor's algorithm, which also relies on the PTM sequence [6].

## 4 Conclusions

We have suggested that self-organization processes of prime integer relations could be used for information processing. For a given problem such processes could be used to efficiently build the correlation structures of a computing system in information processing demonstrating the optimal performance for the problem. Remarkably, the information processing would be distinctive. The processes can be equivalently represented by transformations of two-dimensional geometrical patterns determining the dynamics of the system and revealing its complexity. Also the processes can define the correlation structures without references to the distances, local times and signals between the parts.

Computational experiments testing competitive advantages of the information processing have been presented. They raise the possibility of an optimality condition of complex systems: if the structural complexity of a system is in a certain relationship with the structural complexity of a problem, then the system demonstrates the optimal performance for the problem.

Importantly, the experiments also indicate that the performance of a complex system may behave as a concave function of the structural complexity. Therefore, once the structural complexity could be controlled as a single entity, the optimization of a complex system would be potentially reduced to a one-dimensional concave optimization irrespective of the number of variables involved its description. This might open a way to a new type of information processing for efficient management of complex systems.

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